$Cov(aX,bY)=ab\,Cov(X,Y)$ $\quad Cov(X,Y)=E(XY)-E(X)E(Y)$

$Cov(X,X)=Var(X)$ $\quad Cov(X+a,Y+b)=Cov(X,Y)$ $\quad Corr=\frac{Cov}{\sigma_X\sigma_Y}$

$Var(X+Y)=Var(X)+Var(Y)+2Cov(X,Y)$ $\quad -1\le Corr\le 1$

$-\sigma_X\sigma_Y\le Cov\le\sigma_X\sigma_Y$

$\boxed{Cov(X+Y,W+Z)=Cov(X,W)+Cov(Y,W)+Cov(X,Z)+Cov(Y,Z)}$

$e^x=\sum_{k=0}^{\infty}x^k/k!=\lim_{n\to\infty}(1+x/n)^n$

$\phi_X=E(e^{tX})=\int_x e^{tx}f(x)\,dx$

$E(X^r)=\frac{d^r}{dt^r}\phi_X(t=0)$

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$\Sigma\exp$

$\Gamma(1/2)=\sqrt{\pi}$

Gamma: $f(x)=\frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}$ $\quad \Gamma(\alpha)=\int_0^\infty y^{\alpha-1}e^{-y}dy=(\alpha-1)\Gamma(\alpha-1)$

$\alpha,\beta,x>0$ $\quad E(X)=\alpha\beta$ $\quad Var(X)=\alpha\beta^2$ $\quad \phi_X=1/(1+\beta t)^\alpha$

Gamma: $\mu_r'=\frac{\beta^r\Gamma(\alpha+r)}{\Gamma(\alpha)}$

$\mu_r'=E(X^r)$

$\mu_r=E((X-\mu_1)^r)$

$\mu_0'=1$

$\mu_0=1$

$\mu_1=0$

$\mu_2=Var(X)$

Exp: $f(x)=\frac{1}{\lambda}e^{-x/\lambda}$ $\quad \phi_X=\frac{1}{1-\lambda t}$ if $t-1/\lambda<0$

$x,\lambda>0$ $\quad E(X)=\lambda$ $\quad Var(X)=\lambda^2$

If $Y=aX+b$

$M_Y(t)=e^{tb}M_X(at)$

Special case of gamma $\alpha=\nu/2$ $\beta=2$

Chi: $f(x)=\frac{1}{2^{\nu/2}\Gamma(\nu/2)}x^{\frac{\nu-2}{2}}e^{-x/2}$ $\quad \sum_{k=1}^{n}Z_k^2=\chi^2_{\nu=n}$ $\quad Z$ iid

$x,\nu>0$ $\quad E(X)=\nu$ $\quad Var(X)=2\nu$ $\quad \phi_X=(1-2t)^{-\nu/2}$

If X&Y Indep,

$Z=X+Y$,

$M_Z=M_XM_Y$

Norm: $f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac12\left(\frac{x-\mu}{\sigma}\right)^2}$ $\quad \phi_X=e^{t\mu+\frac12t^2\sigma^2}$

If $X_1,\dots X_n$ iid normal rvs then,

a. $\bar{x}$ and s^2 are indep

b. $(n-1)s^2/\sigma^2$ is $\chi^2_{\nu=n-1}$

Geo: $f(x)=p(1-p)^{x-1}$ $\quad \phi_X=\frac{pe^t}{1-(1-p)e^t}$ $\quad E(X)=1/p$ $\quad Var(X)=\frac{1-p}{p^2}$

Bin: $f(x)=\binom{n}{x}p^x(1-p)^{n-x}$ $\quad \phi_X=(pe^t+1-p)^n$ $\quad E(X)=np$ $\quad Var(X)=np(1-p)$

Markov: $P(X\ge a)\le\frac{E(X)}{a}$ $\quad x\ge0$

Chebyshev: $P(|X-\mu_X|\ge k)\le\frac{\sigma_X^2}{k^2}$ $\quad k>0$

$\bar{X}=\frac{\sum x_i}{n}$

$S^2=\frac{\sum(x_i-\bar{x})^2}{n-1}$

$E(\bar{X})=\mu_X$ $\quad Var(\bar{X})=\frac{\sigma_X^2}{n}$

Order: $g_r(y_r)=\frac{n!}{(r-1)!(n-r)!}\left[\int_{-\infty}^{y_r}f(x)dx\right]^{r-1}f(y_r)\left[\int_{y_r}^{\infty}f(x)dx\right]^{n-r}$

Median: $n=2m+1$, $\tilde{x}=Y_{m+1}$

$h(\tilde{x})=\frac{(2m+1)!}{m!m!}\left[\int_{-\infty}^{\tilde{x}}f(x)dx\right]^m f(\tilde{x})\left[\int_{\tilde{x}}^{\infty}f(x)dx\right]^m$

$n=2m$, $\tilde{x}=\frac12(Y_m+Y_{m+1})$

Mean has less variance than median

$P(\mu-c\le\bar{X}\le\mu+c)\ge1-\frac{\sigma_X^2}{nc^2}$

$Z=\frac{\bar{X}-\mu}{\sigma_X/\sqrt{n}}$

$\lim_{n\to\infty}M_Z=e^{\frac12t^2}$

$\int u\,dv=uv-\int v\,du$

$\lim_{x \to 0} -\ln(x) - 1/x$ is $-\infty$

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Method of moments: r-th sample moment about origin: $M_r' = \frac{\sum x_i^r}{n}$, $M_1' = \bar{x}$, $M_2' = \frac{x_1^2 + \ldots + x_n^2}{n}$

By SL of LN, $M_r' \to E[X^r]$, thus set $M_r' = \mu_r'$ for estimators

MLE: $L(\theta) = f(x_1, x_2, \ldots, x_n; \theta)$ Take derivative and set to 0. $d/d\theta$ not d/dx
$\ln(L(\theta))$ maybe easier to calculate.

USE HATS

SL: $\lim_{n \to \infty} \sum X/n = \mu$ with prob 1

CLT: $\lim_{n \to \infty} \text{Prob}\left(\frac{\sum(X_i - \mu_i)}{\sqrt{\sum \sigma_i^2}} \leq a\right) = \Phi(a)$

WL: $P\left(\left|\frac{\sum X_i}{n} - \mu\right| \geq \varepsilon\right) \to 0$ as $n \to \infty$